EXTINCTION OF SOLUTIONS FOR QUASI-LINEAR PARABOLIC EQUATIONS

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Introduction

- Barenblatt G. I.
  Баренблатт Г. И. О некоторых неустановившихся движениях жидкости и газа в пористой среде // Прикл. матем. и мех. – 1952.

- Oleinic O. A., Kalashnikov A. S., Zhou Yu Lin
  Олейник О. А., Калашников А. С., Юй-Линь Чжоу Задача Коши и краевые задачи для уравнений типа нестационарной фильтрации // Изв. РАН. – 1958.

- Kalashnikov A. S.
  Калашников А. С. О возникновении особенностей у решений уравнения нестационарной фильтрации // Журнал вычисл. матем. и математ. физики.– 1967.

Introduction

Specific properties of non-linear equations include: inertia, inverse movement of front, the localization (strong and weakened) of solutions' supports, extinction of solutions in a finite time and so on.

The qualitative properties of solutions were studied by V. Kondratiev, G. Iosif'yan, E. Radkevich, Brezis H., Kersner R., Friedman A., L.A. Peletier, Diaz J.I., Herrero M.A., Vazquez J.L., Knerr B., Bernis F., Veron L., Shishkov A., Tedeev A., Vespri V., Andreucci D. ....

The most important aspect of such investigations is the description of structural conditions affecting the appearance and disappearance of various non-linear phenomena.

Introduction

Investigations are devoted to the study of the extinction–property of solutions in a finite time to initial-boundary value problems for a wide classes of nonlinear parabolic equations of the second and higher orders with a degenerate absorption potential $a(x)$, whose presence plays a significant role for the mentioned nonlinear phenomena.

EXTINCTION OF SOLUTION

Solution of the problem has the extinction (vanishes) in a finite time if $\exists\ 0 < T^* < \infty$: $u(t,x) = 0$ almost everywhere in $\Omega \quad \forall\ t \geq T^*$.

The questions of a detailed characterization of the effect of extinction of a solution (estimates of the extinction time, asymptotic behavior near the extinction time, etc.) for various classes of semilinear parabolic equations of the diffusion–absorption type were studied in many works:

- **L.E. Payne** //Society for Industrial and Appl. Math. (1975),
- **B.F. Knerr** // Trans. Amer. Math. Soc. (1979),
- **B. Straughan** // Research Notes in Mathematics (1982),
- **C. Bandle, I. Stakgold** // Trans. Amer. Math. Soc. (1984),
- **A. Friedman, M.A. Herrero** // J. Math. Anal. Appl. (1987),
- **Chen Xu-Yan, H. Matano, M. Mimura** // J. Reine Angew. Math. (1995)
- **and etc.**

Brief history of the extinction–property

Phenomenon of extinction was discovered by E. S. Sabinina //Докл. Ан СССР. – 1962. – Т. 143. – №. 4. – С. 794-797 when she studied the first boundary-value problem in a bounded domain with zero boundary data for

$$u_t - (\varphi(u))_{xx} = 0, \qquad N = 1; \tag{1}$$

where $\varphi \in C^\infty(\mathbb{R} \setminus \{0\}) \cap C(\mathbb{R})$, $\varphi(0) = 0, \varphi'(s) > 0$ для $s \neq 0, \varphi'(\pm 0) = +\infty$. Namely for the equation of fast diffusion, it was proved by E. S. Sabinina that the following condition

$$\int\limits_0^1 \frac{ds}{\varphi(s)} < +\infty \tag{2}$$

is necessary and sufficient for the extinction of a solution in a finite time.

Brief history of the extinction–property

For the equation of nonlinear diffusion with absorption

$$u_t - (\varphi(u))_{xx} + \psi(u) = 0$$

in a half-plane $\mathbb{R}^2_+ = \{(t,x) :\ 0 < t < +\infty,\ x \in \mathbb{R}^1\}$, where the functions $\varphi(u) \geq 0$, $\psi(u) \geq 0$ are defined and are continuous for $u \geq 0$ with the initial data

$$u(0,x) = u_0(x) \geq 0, \quad x \in \mathbb{R}^1, \quad u_0 \in C(\mathbb{R}^1) \cap L^\infty(\mathbb{R}^1)$$

A. S. Kalashnikov [О характере распространения тепла в нелинейных средах с существенно нестационарными свойствами// Вестн. МГУ, 1985, №4] proved the sufficiency of the condition

$$\int_0^1 \frac{ds}{\psi(s)} < +\infty \tag{3}$$

for the full cooling (the extinction of a solution) in a finite time.

The phenomenon of vanishing of a solution for semilinear parabolic equations of the diffusion– absorption type

$$u_t - \Delta u + \psi(u) = 0 \quad \text{with nondegenerate potential}$$

was also studied in papers

- **R. Kersner and F. Nicolosi** // J. Math. Anal. Appl., (1992)
- **A. S. Kalashnikov** //Mat. Appl., (1997)
- **Li Jun-Jie**//J. Math. Anal. Appl., (2003)
- **Li Jun-Jie**//J. of Differ. Equa., (2004) and etc.

This type of equation

$$u_t - \Delta u + a(x)|u|^{\lambda-1}u = 0$$

is a simple model to understand some phenomenological properties of nonlinear heat conduction with non-constant strong absorption term.

Brief history of the extinction–property: transition from $u_t - \Delta u + \psi(u) = 0$ to $u_t - \Delta u + a(x)|u|^{\lambda-1}u = 0$

It is well-known (works of J. Diaz, L. Veron, S. Antontsev, S. Shmarev) that if $a(x) \geq c > 0$ on Ω the comparison principle with the solution of the corresponding ordinary equation $u_t + c|u|^{\lambda-1}u = 0, u(0) = \|u_0\|_{L_\infty}$ implies that $u(x,t)$ vanishes for $t \geq T^* = \frac{\|u_0\|_{L_\infty}^{1-\lambda}}{c(1-\lambda)}$.

On the opposite (see papers of M. Cwikel, L. Evans, B. Gidas), if $a(x) \equiv 0$ for any x from some connected open subset $\omega \subset \Omega$, then any solution $u(x,t)$ is bounded from below by

$$\sigma \exp(-t\lambda_\omega)\varphi_\omega(x) \qquad \text{on} \qquad \omega \times (0,\infty),$$

where $\sigma = \operatorname{ess\,inf}_\omega u_0 > 0$, λ_ω and φ_ω are first eigenvalue and corresponding eigenfunction of $-\Delta$ in $W_0^{1,2}(\omega)$.

Extinction of solution for $u_t - \Delta u + a_0(x)|u|^{\lambda-1}u = 0$

V. A. Kondratiev and L. Veron [Asymptotic Analysis, **14**, 1997] were the first who started to investigate the conditions of extinction in a finite time of a solution of the Neumann problem for a semilinear parabolic equation in the case of general potential $a_0(x) \geq 0$. So, they found the sufficient condition which guarantee the extinction in a finite time for the equation with degenerate absorption potential :

$$u_t - \Delta u + a_0(x)|u|^{\lambda-1}u = 0 \quad \text{in } (0,+\infty)\times\Omega,$$

here $a_0(x) \geq 0$: $\inf\limits_{x\in\Omega} a_0(x) = 0$, $0 < \lambda < 1$, Ω is a bounded domain.

This condition takes the form: $\sum\limits_{n=0}^{\infty} \mu_n^{-1} \ln \mu_n < \infty$, where

$$\mu_n = \inf\left\{ \int\limits_{\Omega} (|\nabla\psi|^2 + 2^n a_0(x)\psi^2)\, dx : \psi \in W^{1,2}(\Omega),\ \int\limits_{\Omega} \psi^2 dx = 1 \right\}$$

$n \in \mathbb{N}$.

Starting from previous condition, in **Y. Belaud, B. Helffer, L. Véron** [*Long-time vanishing properties of solutions of sublinear parabolic equations and semi-classical limit of Schrödinger operator//* Ann. Inst. Henri Poincaré Anal. nonlinear, **18**, (2001), N 1, 43–68.] **an explicit sufficient condition** (in the term of potential $a_0(x)$) **of extinction** of a solution for the Cauchy–Neumann problem of the mentioned equation **was obtained**:

- **if** $a_0(x) \geq a_\alpha(|x|) := \exp(-\frac{1}{|x|^\alpha})\ \forall\, x \in \Omega$, $0 < \alpha < 2$, then **problem has extinction in a finite time**;
- in the case $\alpha > 2$ the effect of **vanishing of a solution is lacking**.

The method of investigation which was proposed by Y. Belaud, B. Helffer, L. Véron in *[Ann. Inst. Henri Poincarré Anal. nonlinear, (2001)]* **exploits** estimates of spectrum of the Schrödinger operator. It was also assumed that the solution possesses a certain regularity. In particular, **the exact upper estimates of** $\|u(t,x)\|_{L_\infty(\Omega)}$ were used.

Unfortunately, such an estimate is difficult to obtain or is unknown for solutions of equations of more general structure than the considered above:

$$u_t - \Delta u + a_0(x)|u|^{\lambda-1}u = 0.$$

Extinction of solution for $u_t - \Delta u + a_0(x)|u|^{\lambda-1}u = 0$

In paper Y. Belaud, A. Shishkov *[Long-time extinction of solutions of some semilinear parabolic equations]* // JDE, **238**, (2007), 64–86

$$u_t - \Delta u + a_0(x)|u|^{\lambda-1}u = 0 \quad \text{in } (0, +\infty) \times \Omega \subset \mathbb{R}^N, \; N \geq 1$$

was proposed the technique of local energy estimates, which uses no "additional" properties of regularity of solutions.

Modifying the local energy method in *[Stiepanova K.//UMJ (2014)]* **we proved the extinction in a finite time to the parabolic equation with double nonlinearity and a degenerate absorption term**.

The problem formulation

[Stiepanova K.//UMJ, V.66, № 1. (2014)]

Let Ω be a bounded domain in $\mathbb{R}^N (N \geqslant 1)$ with C^1-boundary, $0 \in \Omega$. Our aim is to investigate the extinction property of solution for initial-boundary problem to a wide class of quasi-linear parabolic equations with the model representative:

$$\left(|u|^{q-1}u\right)_t - \sum_{i=1}^{N} \frac{\partial}{\partial x_i}\left(|\nabla_x u|^{q-1}\frac{\partial u}{\partial x_i}\right) + a_0(x)|u|^{\lambda-1}u = 0 \quad \text{in} \quad Q, \tag{4}$$

$$\left.\frac{\partial u}{\partial n}\right|_{[0,+\infty)\times\partial\Omega} = 0, \tag{5}$$

$$u(0,x) = u_0(x), \quad x \in \Omega. \tag{6}$$

Here $0 \leq \lambda < q$, $a_0(x)$ is a continuous nonnegative function, $Q = (0,+\infty) \times \Omega$ and $u_0(x) \in L_{q+1}(\Omega)$.

Extinction in a finite time to the parabolic equation $\left(|u|^{q-1}u\right)_t - \sum_{i=1}^{N} \frac{\partial}{\partial x_i}\left(|\nabla_x u|^{q-1}\frac{\partial u}{\partial x_i}\right) + a_0(x)|u|^{\lambda-1}u = 0$

Let $0 \in \Omega$ and for an arbitrary absorption potential of mentioned above equation there exists the radially symmetric minorant:

$$a_0(x) \geq d_0 \exp\left(-\frac{\omega(|x|)}{|x|^{q+1}}\right) \quad \forall\, x \in \Omega, \quad d_0 = const > 0, \qquad (7)$$

where $\omega(\cdot) \in C([0, +\infty))$ and nondecreasing function, which satisfies the conditions:

$(A)\, \omega(\tau) > 0 \quad \forall \tau > 0, \quad \omega(0) = 0,$

$(B)\, \omega(\tau) = \omega_0 = const < \infty \quad \forall \tau \in \mathbb{R}^1_+,$

$(C)\, \frac{\tau\,\omega'(\tau)}{\omega(\tau)} < q + 1 - \delta \qquad \forall \tau \in (0, \hat{\tau}), \qquad 0 < \delta < q + 1.$

Main result *[Stiepanova K.//UMJ, V.66, № 1. (2014)]*

under assumption, that:

$$a_0(x) \geq d_0 \exp\left(-\frac{\omega(|x|)}{|x|^{q+1}}\right) \forall x \in \Omega, \;\; d_0 = const > 0 \qquad (7)$$

Theorem.
Let $0 \leq \lambda < q$ *in equation* (4), $u_0(x) \in L_{q+1}(\Omega)$, *let function* $\omega(\cdot)$ *from* (7) *satisfies assumptions* (A), (B), (C) *and the following main condition:*

$$\int\limits_{0+} \frac{\omega(\tau)}{\tau} d\tau < \infty.$$

Then an arbitrary solution $u(t,x)$ *of the problem* (4)–(6) *vanishes on* Ω *in a finite time.*

Extinction of solutions

Further development of investigations in this direction became possible thanks to the work [Y. Belaud, A. Shishkov *Extinction of solutions of some semilinear higher order parabolic equations with degenerate absorption potential* // JDE, **10**, (2010), N 4, 857-882], where an essentially new variant of semi–classical analysis was proposed.

On the contrary with method of **B. Helffer, L. Véron** in the paper **[Ann. Inst. Henri Poincarré Anal. nonlinear, (2001)]** Y. Belaud and A. Shishkov consider a family of first eigenvalues of non-linear Schrödinger operator directly connected with equation, instead of eigenvalues μ_n of auxiliary linear Schrödinger operator, where

$$\mu_n = \inf\left\{ \int\limits_{\Omega} (|\nabla\psi|^2 + 2^n a_0(x)\psi^2)\, dx : \psi \in W^{1,2}(\Omega),\ \int\limits_{\Omega} \psi^2 dx = 1 \right\}$$

$$n \in \mathbb{N}.$$

Extinction of solutions

More precisely in [Y. Belaud, A. Shishkov, JDE, 2010] the family *of the first eigenvalues of a nonlinear Schrödinger operator related to equation*

$$u_t + (-1)^m \sum_{|\eta|=m} D_x^\eta\big(\big|D_x^m u\big|\big) + a(x)|u|^{\lambda-1}u = 0, \quad 0 < \lambda < 1;$$

was considered and sufficient conditions of extinction-property in a finite time were obtained. The estimates of eigenvalues were obtained with the use of the technique of Sobolev embeddings.

Developing, adapting and modifying the semi-classical technique of cited paper [Belaud & Shishkov, JDE, 2010] in *[Stiepanova K.//JMS (2015)]* **we obtained sufficient conditions, which guarantee the extinction in a finite time for more general equation**.

The problem formulation [*Extinction of solutions of higher order parabolic equations with double nonlinearity and degenerate absorption potential*//J. of Math. Sciences, № 3 (2015)]

Let Ω be a bounded domain in $\mathbb{R}^N$, $N \geqslant 1$, with boundary $\partial\Omega$. In the semibounded cylinder $Q = (0, +\infty) \times \Omega$ we consider the following Cauchy–Dirichlet problem:

$$\left(|u|^{q-1}u\right)_t + (-1)^m \sum_{|\eta|=m} D_x^\eta\left(|D_x^m u|^{q-1} D_x^\eta u\right) + a(x)|u|^{\lambda-1}u = 0, \tag{8}$$

$$D_x^\eta u\big|_{(0,+\infty)\times\partial\Omega} = 0 \qquad \forall\, \eta : |\eta| \leq m-1, \tag{9}$$

$$u(0,x) = u_0(x), \qquad x \in \Omega. \tag{10}$$

Here, $m \geq 1, 0 \leq \lambda < q$, $a(x)$ — is nonnegative measurable bounded function in Ω function, and $u_0(x) \in L_{q+1}(\Omega)$.

Main result *[K. Stiepanova//J. of Math. Sci., № 3 (2015)]*

Theorem. Let $N \geq 1,\ m \geq 1,\ 0 \leq \lambda < q$.

(i) If $N < m(q+1)$ and

$$\int\limits_0^c \frac{meas\{x \in \Omega : a(x) \leq s\}}{s}\, ds < +\infty \quad \forall\, c > 0,$$

holds, then any solution $u(t,x)$ of problem (8)–(10) has the extinction in a finite time.

(ii) If $N > m(q+1)$ and

$$\int\limits_0^c \frac{(meas\{x \in \Omega : a(x) \leq s\})^{\frac{m(q+1)}{N}}}{s}\, ds < +\infty \quad \forall\, c > 0,$$

holds, then any solution $u(t,x)$ of problem (8)–(10) has the extinction in a finite time.

[K. Stiepanova//J. of Math. Sciences, № 3 (2015)]

(iii) If $N = m(q+1)$ and

$$\int_0^c \frac{meas\{x \in \Omega : a(x) \le s\}}{s} \times (-\ln meas\{x \in \Omega : a(x) \le s\})\, ds < +\infty \quad \forall\, c > 0,$$

holds, then any solution $u(t,x)$ of problem (8)–(10) has the extinction in a finite time.

Sketch of the proof

We consider the equation:

$$\left(|u|^{q-1}u\right)_t + (-1)^m \sum_{|\eta|=m} D_x^\eta\left(|D_x^m u|^{q-1} D_x^\eta u\right) + a(x)|u|^{\lambda-1}u = 0.$$

Using $u(t,x)$ as a test function in the integral identity, by integration by parts, we obtain

$$\frac{q}{q+1}\int_\Omega \left(|u(t,x)|^{q+1} - |u(0,x)|^{q+1}\right)\, dx + \\ + \int_0^t \int_\Omega \left(|D_x^m u|^{q+1} + a(x)|u|^{\lambda+1}\right) dx\, dt = 0.$$

Sketch of the proof

Let us introduce the spectral characteristic

$$\lambda_1(h) := \inf\left\{ \int\limits_{\Omega} \left(|D_x^m v|^{q+1} + a(x)|v|^{\lambda+1}\right)\, dx, \right.$$

$$\left. v \in \overset{\circ}{W}{}^m_{q+1}(\Omega), \quad ||v||^{q+1}_{L^{q+1}(\Omega)} = h \right\}.$$

Proposition [Key-Stone]

$$\text{If} \quad \int\limits_0^c \frac{dh}{\lambda_1(h)} < +\infty, \tag{11}$$

then any solution of problem (8)–(10) vanishes in finite time

$$T \leq \frac{q}{q+1} \int\limits_0^{\tilde{c}} \frac{dh}{\lambda_1(h)}, \quad \text{where} \quad \tilde{c} = ||u_0||^{q+1}_{L^{q+1}(\Omega)}.$$

Sketch of the proof

$$\frac{q}{q+1}\int\limits_{\Omega}\left(|u(t,x)|^{q+1}-|u(0,x)|^{q+1}\right)\,dx+$$

$$+\int\limits_0^t\int\limits_{\Omega}\left(|D_x^m u|^{q+1}+a(x)|u|^{\lambda+1}\right)dx\,dt=0.$$

The first term in this equality is absolutely continuous in t and has a derivative almost everywhere. The second term is also absolutely continuous in t. Therefore, by differentiating with respect to t, we obtain, by virtue of definition λ_1, from the last relation that

$$\frac{d}{dt}\left(\|u\|_{L^{q+1}(\Omega)}^{q+1}\right)+\frac{q+1}{q}\,\lambda_1\left(\|u\|_{L^{q+1}(\Omega)}^{q+1}\right)\leq 0,$$

By solving the ordinary differential inequality, we arrive at the key Proposition.

Sketch of the proof

Hence, if we demonstrate the convergence of the integral

$$\int_0^c \frac{dh}{\lambda_1(h)} < ??? < +\infty$$

the effect of extinction of solution will be proved.

Thus, for $v \in \overset{\circ}{W}{}^m_{q+1}(\Omega)$: $\|v\|^{q+1}_{L^{q+1}(\Omega)} = h$ let us consider the functi-onal

$$F(v) = \int_\Omega \left(|D^m_x v|^{q+1} + a(x)|v|^{\lambda+1} \right) dx \rightarrow min.$$

Main result *[K. Stiepanova//J. of Math. Sci., № 3 (2015)]*

Theorem. Let $N \geq 1,\ m \geq 1,\ 0 \leq \lambda < q$.

(i) If $N < m(q+1)$ and

$$\int_0^c \frac{meas\{x \in \Omega : a(x) \leq s\}}{s}\, ds < +\infty \quad \forall\, c > 0,$$

holds, then any solution $u(t,x)$ of problem (8)–(10) has the extinction in a finite time.

(ii) If $N > m(q+1)$ and

$$\int_0^c \frac{(meas\{x \in \Omega : a(x) \leq s\})^{\frac{m(q+1)}{N}}}{s}\, ds < +\infty \quad \forall\, c > 0,$$

holds, then any solution $u(t,x)$ of problem (8)–(10) has the extinction in a finite time.

Sufficient condition for extinction in a finite time case $N \neq m(q+1)$

$$\int_0^c \frac{(meas\{x \in \Omega : a(x) \leq s\})^\theta}{s} ds < +\infty, \quad \theta = \min\left\{\frac{m(q+1)}{N}; 1\right\}$$

Corollary Let $a(x) = \exp\left(-\frac{\omega(|x|)}{|x|^{N\theta}}\right)$, where $\omega(\cdot)$ be defined and continuous on $[0, +\infty)$, be a nondecreasing nonnegative function, such that $\omega(r) \leq \omega_0 = const < \infty \ \forall r \in [0, +\infty)$ and satisfies

$$\int_0^c \frac{\omega(\tau)}{\tau} d\tau < +\infty, \quad c = const > 0.$$

Then any solution $u(t, x)$ of problem (8)–(10) has the extinction in a finite time.

For $\alpha > 0$ relation yields:

$$meas\left\{x \in \Omega : \exp\left(-\frac{\omega(|x|)}{|x|^\alpha}\right) \le s\right\} = meas\left\{|x|^\alpha \le \frac{\omega(|x|)}{-\ln s}\right\}$$

$$= meas\left\{x \in \Omega : |x| \le \left(\frac{\omega(|x|)}{-\ln s}\right)^{\frac{1}{\alpha}}\right\} \le C_N \left(\frac{\omega(|x|)}{-\ln s}\right)^{\frac{N}{\alpha}}.$$

Obviously, that $a(x) \le s \iff \frac{\omega(|x|)}{|x|^\alpha} \ge -\ln s$, by virtue of the conditions on the function ω: $\frac{\omega_0}{|x|^\alpha} \ge -\ln s$, whence

$$|x| \le \left(\frac{\omega_0}{-\ln s}\right)^{\frac{1}{\alpha}}.$$

For $\alpha = N\theta$ we obtain:

$$meas\{x \in \Omega : a(x) \le s\} = meas\left\{x \in \Omega : |x|^{N\theta} \le \frac{\omega\left(\left(\frac{\omega_0}{-\ln s}\right)^{\frac{1}{N\theta}}\right)}{-\ln s}\right\}$$

$$\le C_N \left\{\frac{\omega\left(\left(\frac{\omega_0}{-\ln s}\right)^{\frac{1}{N\theta}}\right)}{-\ln s}\right\}^{\frac{1}{\theta}}.$$

Then the condition of convergence of the integral in Theorem takes the form

$$\int_0^c \frac{(meas\{x \in \Omega : \ a(x) \le s\})^\theta}{s} ds \le \int_0^{1/e} C_N^\theta \frac{\omega\left(\left(\frac{\omega_0}{-\ln s}\right)^{\frac{1}{N\theta}}\right)}{s(-\ln s)} ds$$

$$= \left[\text{Let } y = \frac{\omega_0}{-\ln s} \Rightarrow dy = -\frac{\omega_0}{(-\ln s)^2}\left(-\frac{ds}{s}\right),\right.$$

$$\left.\text{that is} \quad \frac{ds}{s(-\ln s)} = -\ln s \frac{dy}{\omega_0} = \frac{dy}{y}\right]$$

$$= C_N^\theta \int_0^{\omega_0} \frac{\omega\left(y^{\frac{1}{N\theta}}\right)}{y} dy = \left[\text{Let us change the variable } \tau = y^{\frac{1}{N\theta}}, \text{ hence}\right.$$

$$\left. N\theta\, \tau^{N\theta-1} d\tau = dy, \text{ that is } N\theta \frac{d\tau}{\tau} = \frac{dy}{y}\right] = N\theta\, C_N^\theta \int_0^{\omega_0^{\frac{1}{N\theta}}} \frac{\omega(\tau)}{\tau} d\tau < +\infty,$$

which was to be proved.

EXTINCTION OF SOLUTION IN A FINITE TIME

The research of vanishing property was evolved from simpler to more difficult equations according to the lineup:

- $$u_t - u_{xx} + c|u|^{\lambda-1}u = 0,$$
- $$u_t - \triangle u + a_0(x)|u|^{\lambda-1}u = 0,$$
- $$\left(|u|^{q-1}u\right)_t - \sum_{i=1}^{N} \frac{\partial}{\partial x_i}\left(|\nabla_x u|^{q-1}\frac{\partial u}{\partial x_i}\right) + a_0(x)|u|^{\lambda-1}u = 0,$$
- $$u_t + (-1)^m \sum_{|\alpha|=m} D_x^\alpha |D_x^m u| + a(x)|u|^{\lambda-1}u = 0,$$
- $$\left(|u|^{q-1}u\right)_t + (-1)^m \sum_{|\alpha|=m} D_x^\alpha\left(|D_x^m u|^{q-1}D_x^\alpha u\right) + a(x)|u|^{\lambda-1}u = 0.$$

CONCLUSIONS

To sum up, report was devoted to the study of some questions from qualitative theory of solutions to initial-boundary value problems for nonlinear parabolic equations with a degenerate absorption potential.

- Cauchy-Neumann problem to the equation of non–stationary diffusion with homogeneous main part and a degenerate radial absorption potential was studied. Theorem about sufficient condition for extinction in a finite time was obtained.

- Cauchy-Dirichlet problem for quasilinear parabolic equations of higher order with a degenerate absorption potential was investigated. Sufficient conditions, which guarantee the extinction, were obtained.

THANK YOU FOR ATTENTION!