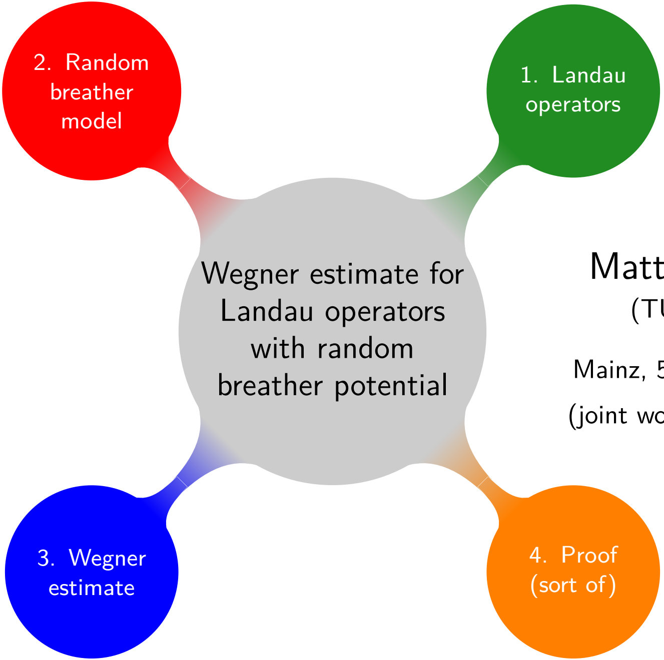
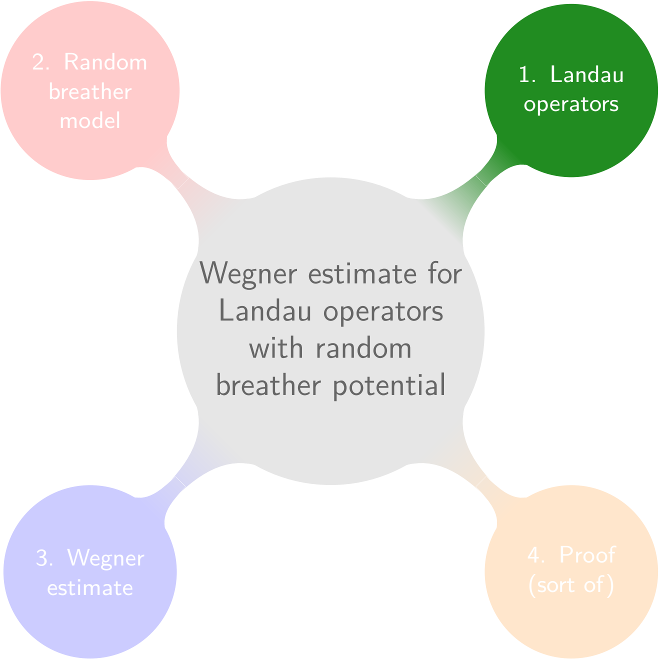


Matthias Täufer
(TU Chemnitz)

Mainz, 5 September 2016

(joint work with I. Veselić)


Wegner estimate for Landau operators with random breather potential
1. Landau operators
2. Random breather model
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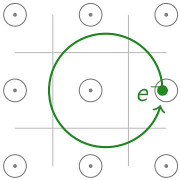

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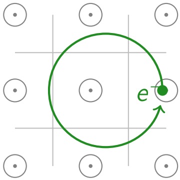

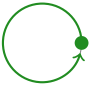

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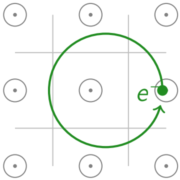

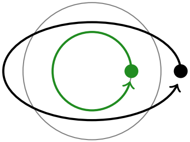

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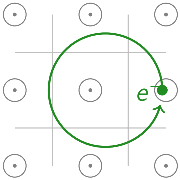

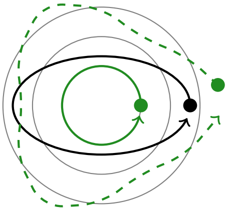

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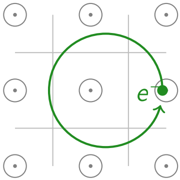

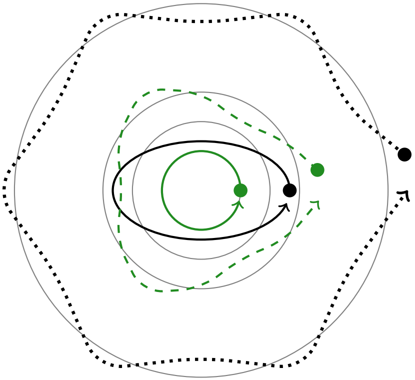

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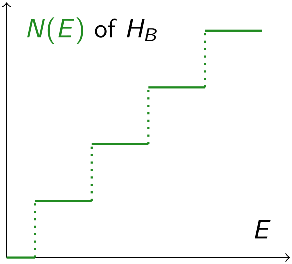

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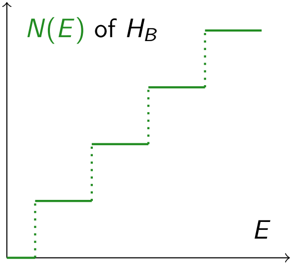

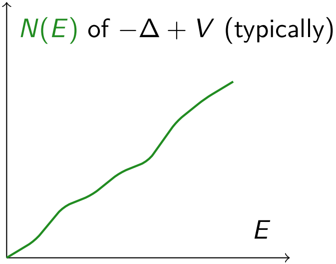

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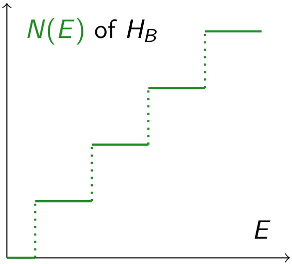

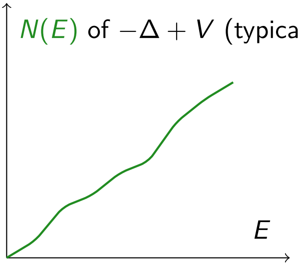

- Jumps in IDS related to *quantum hall effect*; since 1990 SI definition for electric resistance


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Now add random potential

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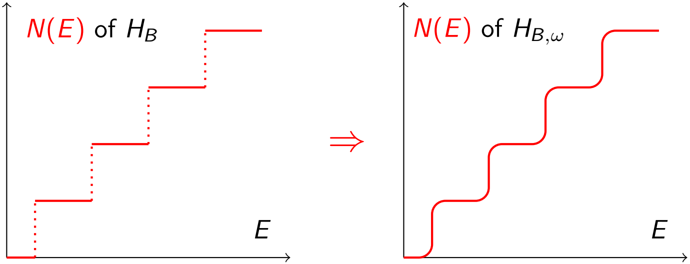

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We are interested in the *random breather potential*

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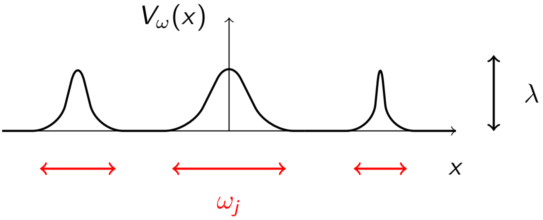

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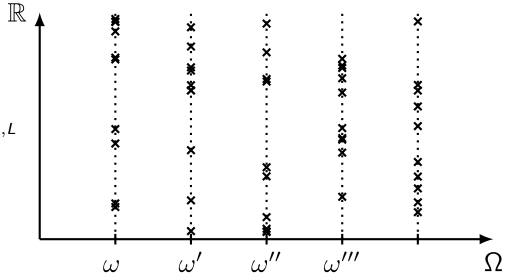
Spectrum of $H_{\omega,L}$



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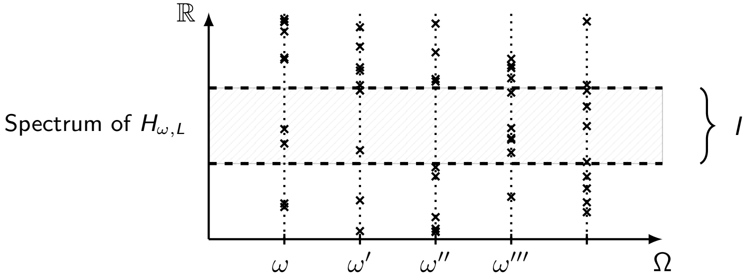

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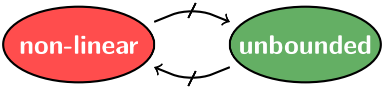

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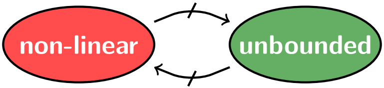

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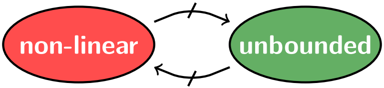

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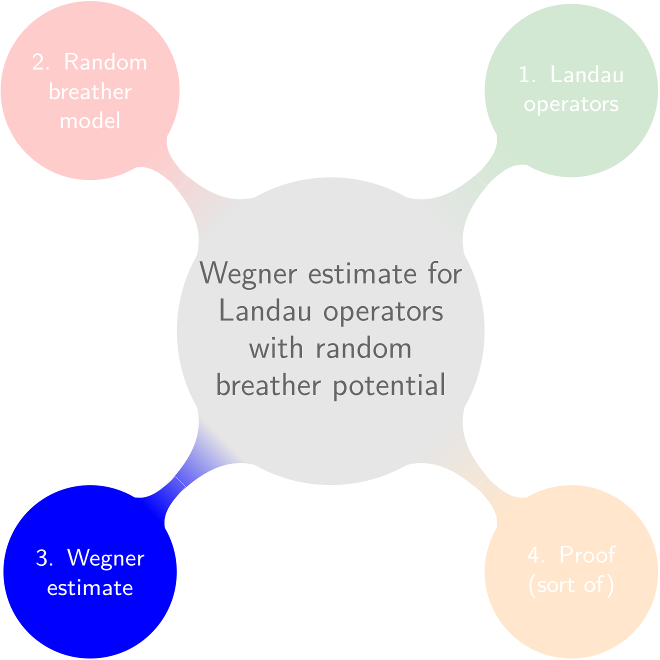
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Theorem (Wegner estimate for Landau operator with random breather potential, T ,Veselić 16)

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Let $B > 0$, $E_0 \in \mathbb{R}$, $\theta \in (0,1)$. *Then there is* $\lambda_0 > 0$ *such that for all* $0 < \lambda < \lambda_0$, *all intervals* $I \subset (-\infty, E_0]$ *and all sufficiently large* L *we have*

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Examples:

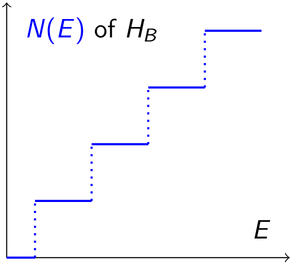
- The hat potential $u(x) = \chi_{|x|\leq 1} \cdot (1 - |x|)$,
- the smooth bump potential $u(x) = \chi_{|x|\leq 1} \cdot \exp(-1/(1 - x^2))$.

3. Wegner estimate

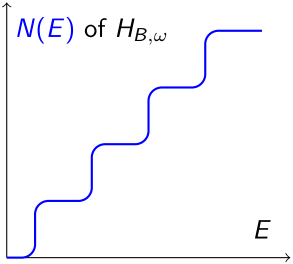
Corollary

The integrated density of states is locally Hölder continuous with respect to every exponent $\theta \in (0, 1)$.

- Physisicts' fact verified:



$\Rightarrow$

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- Decomposition with respect to $\chi_J(H_B)$ à la [Combes, Hislop, Klopp 03/07],
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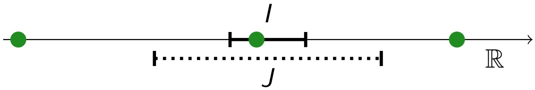
$$= \operatorname{Tr}[\chi_I(H_{B,\omega})\chi_J(H_B)] + \operatorname{Tr}[\chi_I(H_{B,\omega})(\operatorname{Id} - \chi_J(H_B))]$$



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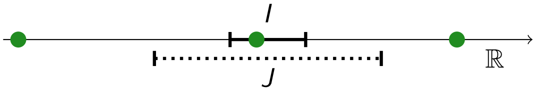

- Second trace: use that I and J^c are far apart

$$\leq \operatorname{Tr}\left[\chi_I(H_{B,\omega})\chi_J(H_B)\right] + \frac{\|\lambda V_\omega\|^2}{\operatorname{dist}(I,J^c)^2} \cdot \operatorname{Tr}\left[\chi_I(H_{B,\omega})\right]$$

4. Proof (sort of)

$$\mathrm{Tr}\left[\chi_I(H_{B,\omega})\right] \leq \mathrm{Tr}\left[\chi_I(H_{B,\omega})\chi_J(H_B)\right] + \frac{\|\lambda V_\omega\|^2}{\mathrm{dist}(I, J^c)^2} \cdot \mathrm{Tr}\left[\chi_I(H_{B,\omega})\right]$$

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$$\mathrm{Tr}\left[\chi_I(H_{B,\omega})\right] \leq \mathrm{Tr}\left[\chi_I(H_{B,\omega})\chi_J(H_B)\right] + \frac{\|\lambda V_\omega\|^2}{\mathrm{dist}(I,J^c)^2} \cdot \mathrm{Tr}\left[\chi_I(H_{B,\omega})\right]$$

- First trace: smuggle in $\sum_j \partial_{\omega_j} V_\omega$ by estimate on $\chi_J(H_B)$, needs that J contains at most one Landau level

$$\leq C\,\mathrm{Tr}\left[\chi_I(H_{B,\omega})\chi_J(H_B)\left(\sum_j \partial_{\omega_j} V_\omega\right)\chi_J(H_B)\right] + \frac{\|\lambda V_\omega\|^2}{\mathrm{dist}(I,J^c)^2} \cdot \ldots$$

4. Proof (sort of)

$$\operatorname{Tr}\left[\chi_I(H_{B,\omega})\right] \leq \operatorname{Tr}\left[\chi_I(H_{B,\omega})\chi_J(H_B)\right] + \frac{\|\lambda V_\omega\|^2}{\operatorname{dist}(I, J^c)^2} \cdot \operatorname{Tr}\left[\chi_I(H_{B,\omega})\right]$$

- First trace: smuggle in $\sum_j \partial_{\omega_j} V_\omega$ by estimate on $\chi_J(H_B)$, needs that J contains at most one Landau level

$$\leq C \operatorname{Tr}\left[\chi_I(H_{B,\omega})\chi_J(H_B)\left(\sum_j \partial_{\omega_j} V_\omega\right)\chi_J(H_B)\right] + \frac{\|\lambda V_\omega\|^2}{\operatorname{dist}(I, J^c)^2} \cdot \ldots$$

- Rearrange and hide $\chi_I(H_{B,\omega})$ on the left hand side to find

$$\operatorname{Tr}\left[\chi_I(H_{B,\omega})\right] \leq C \operatorname{Tr}\left[\chi_I(H_{B,\omega})\left(\sum_j \partial_{\omega_j} V_\omega\right)\right].$$

4. Proof (sort of)

$$\operatorname{Tr}\left[\chi_I(H_{B,\omega})\right] \leq \operatorname{Tr}\left[\chi_I(H_{B,\omega})\chi_J(H_B)\right] + \frac{\|\lambda V_\omega\|^2}{\operatorname{dist}(I,J^c)^2} \cdot \operatorname{Tr}\left[\chi_I(H_{B,\omega})\right]$$

- First trace: smuggle in $\sum_j \partial_{\omega_j} V_\omega$ by estimate on $\chi_J(H_B)$, needs that J contains at most one Landau level

$$\leq C \operatorname{Tr}\left[\chi_I(H_{B,\omega})\chi_J(H_B)\left(\sum_j \partial_{\omega_j} V_\omega\right)\chi_J(H_B)\right] + \frac{\|\lambda V_\omega\|^2}{\operatorname{dist}(I,J^c)^2} \cdot \ldots$$

- Rearrange and hide $\chi_I(H_{B,\omega})$ on the left hand side to find

$$\operatorname{Tr}\left[\chi_I(H_{B,\omega})\right] \leq C \operatorname{Tr}\left[\chi_I(H_{B,\omega})\left(\sum_j \partial_{\omega_j} V_\omega\right)\right].$$

- Requires $\|\lambda V_\omega\| / \operatorname{dist}(I,J^c)$ small. (Here the assumption $\lambda << 1$ enters).

4. Proof (sort of)

These steps are summarized by the following lemma

Lemma (T, Veselić 16)

H lower semibounded, purely discrete spectrum, V bdd. symmetric, $I \subset J \subset \mathbb{R}$ intervals. Assume there is $W \geq 0$ such that

$$\chi_J(H) W \chi_J(H) \geq C \chi_J(H).$$

Then, for $\|V\|$ small we have

$$\operatorname{Tr}[\chi_I(H+V)] \leq \tilde{C} \operatorname{Tr}(\chi_I(H+V)(W+W^2))$$

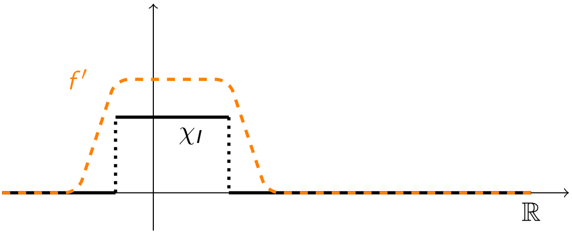
and $\tilde{C}$ is known explicitely

4. Proof (sort of)

$$\mathrm{Tr}\left[\chi_I(H_{B,\omega})\right] \leq C\,\mathrm{Tr}\left[\chi_I(H_{B,\omega})\left(\sum_j \partial_{\omega_j} V_\omega\right)\right]$$

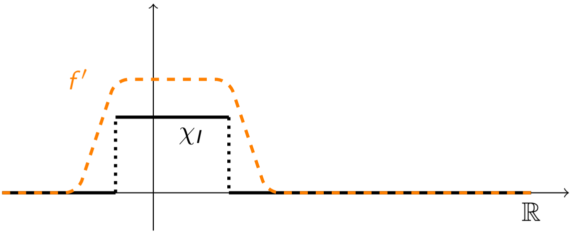
4. Proof (sort of)

$$\begin{aligned}\mathrm{Tr}\left[\chi_I(H_{B,\omega})\right] &\leq C\,\mathrm{Tr}\left[\chi_I(H_{B,\omega})\left(\sum_j \partial_{\omega_j} V_\omega\right)\right] \\ &\leq C\,\mathrm{Tr}\left[f'(H_{B,\omega})\left(\sum_j \partial_{\omega_j} V_\omega\right)\right]\end{aligned}$$



4. Proof (sort of)

$$\begin{aligned}
\mathrm{Tr}\left[\chi_I(H_{B,\omega})\right] &\leq C\,\mathrm{Tr}\left[\chi_I(H_{B,\omega})\left(\sum_j \partial_{\omega_j} V_\omega\right)\right] \\
&\leq C\,\mathrm{Tr}\left[f'(H_{B,\omega})\left(\sum_j \partial_{\omega_j} V_\omega\right)\right] \\
&= C\,\mathrm{Tr}\left[\sum_j \partial_{\omega_j}\left(f(H_{B,\omega})\right)\right]
\end{aligned}$$



4. Proof (sort of)

Take expectation

$$\mathbb{E}\,\mathrm{Tr}\left[\chi_I(H_{B,\omega})\right] \leq C\mathbb{E}\,\mathrm{Tr}\left[\sum_j \partial_{\omega_j}\left(f(H_{B,\omega})\right)\right]$$

4. Proof (sort of)

Take expectation

$$\mathbb{E}\,\mathrm{Tr}\left[\chi_I(H_{B,\omega})\right] \leq C\mathbb{E}\,\mathrm{Tr}\left[\sum_j \partial_{\omega_j}\left(f(H_{B,\omega})\right)\right]$$

Rest of the proof is folklore:

- sum over j gives L^2 (volume)
- probability estimate gives $|I|^\theta$

$$\mathbb{E}\,\mathrm{Tr}\left[\chi_I(H_{B,\omega})\right] \leq C \cdot |I|^\theta \cdot L^2. \quad \square$$

Some References

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Thank you for your attention!
1. Landau operators
2. Random breather model
3. Wegner estimate
4. Proof (sort of)